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A Mode I Crack Problem for a Thermoelastic Fibre-Reinforced Anisotropic Material Using Finite Element Method

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Abstract—In this article, the theory of generalized thermoelasticity with one relaxation time is used to investigate the thermoelastic fiber-reinforced anisotropic material with a finite linear crack. The crack boundary is due to a prescribed temperature and stress distribution. By using the finite element method, the numerical solutions of the components of displacement, temperature and the stress components have been obtained. Comparisons of the results in the absence and presence of reinforcement have been presented.

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1. INTRODUCTION

Fibers are assumed to be an inherent property of matter, rather than some form of inclusion in such models as Spencer [1]. Fiber-reinforced composite materials are widely used in technical structures. These materials have considerably high strength with respect to their weight, even at high temperatures they keep their stiffness. Continuum models are commonly used to explain the mechanical properties of these materials. There are some researches in the field of thermoelastic behavior of fibrous composites, among which two of them are well-known. Firstly, Lord and Shulman [2] presented the generalized thermoelastic theory with one relaxation time by postulating a new law of thermal conduction instead of the classical Fourier law. Secondly, Green and Lindsay [3] presented two relaxation times effects on the generalized thermoelastic theory. Dhaliwal and Sherief [4] extended the generalized thermoelastic theories for the anisotropic medium. The material strength in the presence of cracks is an attracting problem of fracture mechanics and the knowledge of the elastic stress fields is potentially useful for strength estimation based on the available theories for brittle fracture [5–7]. Several researches have been published which treated the stress distributions in an unbounded solid due to the application of normal pressure or temperature on the faces of a circular internal flat crack [8, 9].

The exact solution of the basic equations of generalized thermoelastic models for linear/nonlinear coupled system exists for initial and boundary issues which are very specific and simple cases. Therefore one can choose the finite element method. Basically there are three steps to apply the finite element method. The first step is to take the overall behavior of the variables so as to satisfy the differential equations given unknown field. The second step is temporal integration. The temporal derivatives of the unknown variables must be determined by the previous results. The last step is to solve the resulting equations from the first and second steps by the algorithm of the finite element method [10].

The present paper investigates a Lord and Sherman model in a two-dimensional thermoelastic medium containing a mode I crack. The nondimensional equations have been solved numerically using the finite element method.

2. BASIC EQUATIONS AND FORMULATION OF THE PROBLEM

An infinite space $-\infty < y < \infty$, $-\infty < x < \infty$, containing a crack on the y axis, $|x| \le b$, $x = \pm 0$ was considered for the problem. The crack surface is subjected to a prescribed temperature and normal stress distribution. The preferred direction of the x axis was considered for the fiber direction as $\mathbf{a} \equiv (1, 0, 0)$. All the considered functions depend on x and y with the time t. Thus, the components of displacement vector are $u(x, y, t)$ and $v(x, y, t)$. In this case, the governing equations have the following form [11]:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = \rho \frac{\partial^2 u}{\partial t^2}, \tag{1}$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2}, \tag{2}$$

$$K_{11}\frac{\partial^2 T}{\partial x^2} + K_{22}\frac{\partial^2 T}{\partial y^2} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) \times \left(\rho c_e T + T_0 \gamma_{11}\frac{\partial u}{\partial x} + T_0\gamma_{22}\frac{\partial v}{\partial y}\right), \tag{3}$$

$$\sigma_{xx} = (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)\frac{\partial u}{\partial x} + (\lambda + \alpha)\frac{\partial v}{\partial y} - \gamma_{11}(T - T_0), \tag{4}$$

$$\sigma_{yy} = (\lambda + 2\mu_T)\frac{\partial v}{\partial y} + (\lambda + \alpha)\frac{\partial u}{\partial x} - \gamma_{22}(T - T_0), \tag{5}$$

$$\sigma_{xy} = \mu_L\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right), \tag{6}$$

where

$$\gamma_{11} = (2\lambda + 3\alpha + 4\mu_L - 2\mu_T + \beta)\alpha_{11} + (\lambda + \alpha)\alpha_{22},$$
$$\gamma_{22} = (2\lambda + \alpha)\alpha_{11} + (\lambda + 2\mu_T)\alpha_{22},$$

α_{11}, α_{22} are the linear thermal expansion coefficients, T_0 is the reference uniform temperature, T is the incremental temperature, K_{11} and K_{22} are the thermal conductivity components, ρ is the mass density, c_e is the specific heat at constant strain, λ and μ_T are the elastic constants, τ_{xx}, τ_{xy} and τ_{yy} are the stress components, α, β, $(\mu_L - \mu_T)$ are the elastic parameters of a fiber reinforced material. We will use the nondimensional form of the previous equations. The nondimensional parameters are

$$(\sigma'_{xx}, \sigma'_{xy}, \sigma'_{yy}) = \frac{1}{m}(\sigma_{xx}, \sigma_{xy}, \sigma_{yy}),$$

$$(t', \tau'_0) = \frac{c^2(t, \tau_0)}{\eta} T' = \frac{T - T_0}{T_0},$$
$$(x', y', u', v') = \frac{c}{\eta}(x, y, u, v), \tag{7}$$

where

$$\eta = K_{11}/(\rho c_e),\ c^2 = m/\rho,$$
$$m = \lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta.$$

With respect to the nondimensional quantities in Eq. (7), after neglecting the primes for convenience, the previous equations reduced to

$$\frac{\partial^2 u}{\partial x^2} + s_1\frac{\partial^2 v}{\partial x \partial y} + s_2\frac{\partial^2 u}{\partial y^2} - s_3\frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{8}$$

$$s_4\frac{\partial^2 v}{\partial z^2} + s_1\frac{\partial^2 u}{\partial x \partial y} + s_2\frac{\partial^2 v}{\partial x^2} - s_5\frac{\partial T}{\partial y} = \frac{\partial^2 v}{\partial t^2}, \tag{9}$$

$$\frac{\partial^2 T}{\partial x^2} + s_6\frac{\partial^2 T}{\partial y^2} = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\left[T + s_7\frac{\partial u}{\partial x} + s_8\frac{\partial v}{\partial y}\right], \tag{10}$$

$$\sigma_{xx} = \frac{\partial u}{\partial x} + (s_1 - s_2)\frac{\partial v}{\partial y} - s_3 T, \tag{11}$$

$$\sigma_{yy} = (s_1 - s_2)\frac{\partial u}{\partial x} + s_4\frac{\partial v}{\partial y} - s_5 T, \tag{12}$$

$$\sigma_{xy} = s_2\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{13}$$

where

$$s_1 = \frac{\alpha + \lambda + \mu_L}{m},\ s_2 = \frac{\mu_L}{m},\ s_3 = \frac{\beta_{11}T_0}{m},\ s_4 = \frac{\lambda + 2\mu_T}{m},$$
$$s_5 = \frac{\beta_{22}T_0}{m},\ s_6 = \frac{K_{22}}{K_{11}},\ s_7 = \frac{\gamma_{11}}{\rho c_e},\ s_8 = \frac{\gamma_{22}}{\rho c_e}.$$

To solve this problem, the boundary and initial conditions must be considered. The initial conditions are the following:

$$T(x, y, 0) = \frac{\partial}{\partial t}T(x, y, 0) = 0,$$
$$u(x, y, 0) = \frac{\partial}{\partial t}u(x, y, 0) = 0, \tag{14}$$
$$v(x, y, 0) = \frac{\partial}{\partial t}v(x, y, 0) = 0.$$

At $x = 0$, the boundary conditions are assumed as (Fig. 1)

$$u = 0,\ |y| > b, \tag{15}$$

$$\frac{\partial T}{\partial x} = 0,\ |y| > b, \tag{16}$$

$$T = T_1 H(t),\ |y| \le b, \tag{17}$$

$$\sigma_{xx} = -P_1 H(t),\ |y| \le b, \tag{18}$$

$$\sigma_{xy} = 0,\ -\infty < y < \infty, \tag{19}$$